

Bosonic sector of $D = 11$ superstring action and the critical dimension. Toy model.

A.A. Deriglazov*

Instituto de Física, Universidade Federal do Rio de Janeiro,
Rio de Janeiro, Brasil.

Abstract

Bosonic model inspired by $D = 11$ superstring action is investigated. An appropriate set of variables is found, in which the light-cone quantization turns out to be possible. It is shown that anomaly terms in the algebra of the light-cone Poincare generators are absent for the case $D = 27$.

Construction of $D = 11$ Green-Schwarz type superstring action presents a nontrivial problem already at the classical level. The reason is that only for the dimensions $D = 3, 4, 6, 10$ the action is invariant under the local κ -symmetry (as well as under the global supersymmetry) [1]. Recently it was recognized [2-6] that the problem can be resolved if one introduces an additional vector variable n^N into the formulation. The corresponding $D = 11$ action (which incorporates $n^N(\tau, \sigma)$ as the dynamical variable) was suggested in [3]. Similarly to the Green-Schwarz construction, the action has κ -symmetry which allows one to remove half of fermionic coordinates and supply free dynamics for the physical variables as well as the discrete mass spectrum [3,4]. Moreover, n^N -independent part of spectrum (being classified with respect to $SO(1, 9)$ group) was identified with the type IIA superstring states. For the massless level classified with respect to $SO(1, 10)$ group one gets the supergravity multiplet in $D = 11$ [7-9]. The other states (presented on each mass level) may correspond to the states of the uncompactified M-theory limit [9,10]. Due to these properties one hopes that such a kind theory can be reasonable extension of the Green-Schwarz action to the case $D = 11$.

The aim of this work is to investigate some quantum properties of the theory. It will be demonstrated that light-cone quantization of the bosonic sector is possible (the corresponding Lagrangian will be discussed below), which allows one to compute algebra of the

*alexei@fisica.ufjf.br On leave of absence from Dept. Math. Phys., Tomsk Polytechnical University, Tomsk, Russia

light-cone Poincare generators. We show that anomaly terms in the algebra are absent for the case $D = 27$. Fermionic sector of $D = 11$ superstring action consist of $D = 11$ Majorana spinor which can be decomposed on a pair of the Majorana - Weyl spinors of an opposite chirality with respect of $SO(1, 9)$ group. From this fact and from the result $D = 27$ for the bosonic sector one expects that the critical dimension for the superstring presented in [3,4] is $D = 11$.

Let π^N is zero mode of $n^N(\tau, \sigma)$ [5], and the corresponding canonically conjugated variable will be denoted as \tilde{y}^N . Our starting point is the Virasoro constraints

$$L_n = \frac{1}{2} \sum_{\forall k} \tilde{\alpha}_{n-k}^N \tilde{\alpha}_k^N = 0, \quad \bar{L}_n = \frac{1}{2} \sum_{\forall k} \tilde{\tilde{\alpha}}_{n-k}^N \tilde{\tilde{\alpha}}_k^N = 0, \quad (1)$$

accompanied by the first class constraint $\pi^N \pi^N = -\varepsilon = \text{const}$ and by the following second class system

$$\pi^N \tilde{\alpha}_n^N = 0, \quad \pi^N \tilde{\tilde{\alpha}}_n^N = 0, \quad n \neq 0; \quad (2)$$

$$\pi^N \tilde{\alpha}_0^N = 0, \quad \pi^N \tilde{x}^N = 0. \quad (3)$$

Below we will omit expressions for the left moving oscillators $\tilde{\alpha}^N$. The cases of $SO(1, D-1)$ and $SO(2, D-2)$ group will be considered simultaneously: $\eta^{NM} = (\eta^{\mu\nu}, \eta^{D-1, D-1} \equiv \eta)$, $\eta = \pm 1$, $\eta^{\mu\nu} = (-, +, \dots, +)$, $\mu, \nu = 0, 1, \dots, D-2$. The parameters ε, η are not fixed (except the restrictions which follows from the constraints) throughout the work, but is expected to be fixed in the supersymmetric version [4]. The string tension is chosen to be $T = \frac{1}{4\pi}$ such that $\tilde{\alpha}_0^N = -\tilde{\tilde{\alpha}}_0^N = \tilde{p}^N$. The system (1), (2) can be obtained by means of partial fixation of gauge for the bosonic constraints presented in the theory [6]. As it was shown in [3,4], these constraints (and the corresponding terms in the action) are essential for establishing of the κ -symmetry. Below we present also an action which leads to the complete system (1)-(3).

D -dimensional Poincare generators are realized as

$$\mathbf{P}^N = -\tilde{p}^N, \quad \mathbf{J}^{MN} = \tilde{x}^{[M} \tilde{p}^{N]} + iS^{MN} + i\bar{S}^{MN} + \tilde{y}^{[M} \pi^{N]},$$

$$S^{MN} \equiv \sum_{n=1}^{\infty} \frac{1}{n} \tilde{\alpha}_{-n}^{[M} \tilde{\alpha}_n^{N]}. \quad (4)$$

From Eq.(2) it follows that one component of each oscillator is gauge degree of freedom. So one expects that only the remaining $D-1$ components will give contribution into the anomaly terms, such that the condition of absence of the anomaly will be: $D-1 = 26$. We support this suggestion by direct calculations.

To quantize the theory we follow to the standard prescription [11,12]. The second class constraints (2), (3) can be taken into account by means of introduction of the corresponding Dirac bracket. The non zero brackets for our basic variables turn out to be

$$\begin{aligned}
\{\tilde{x}^N, \tilde{p}^M\} &= \Pi^{NM} \equiv \eta^{NM} - \frac{1}{\pi^2} \pi^N \pi^M, \\
\{\tilde{\alpha}_n^N, \tilde{\alpha}_k^M\} &= in \delta_{n+k,0} \Pi^{NM}, \\
\{\tilde{y}^N, \pi^M\} &= \eta^{NM}, \\
\{\tilde{y}^N, \tilde{y}^M\} &= -\frac{1}{\pi^2} \tilde{x}^{[N} \tilde{p}^{M]} - i \sum_{n=1}^{\infty} \frac{1}{n\pi^2} (\tilde{\alpha}_{-n}^{[N} \tilde{\alpha}_n^{M]} + \tilde{\alpha}_{-n}^{[N} \tilde{\alpha}_n^{M]}), \\
\{\tilde{x}^M, \tilde{y}^N\} &= \frac{1}{\pi^2} \pi^M \tilde{x}^N, \quad \{\tilde{p}^M, \tilde{y}^N\} = \frac{1}{\pi^2} \pi^M \tilde{p}^N, \\
\{\tilde{\alpha}_n^M, \tilde{y}^N\} &= \frac{1}{\pi^2} \pi^M \tilde{\alpha}_n^N,
\end{aligned} \tag{5}$$

and the same expressions for the left moving oscillators $\tilde{\alpha}_n^N$. Now Eqs.(2),(3) can be solved

$$\tilde{z}^{D-1} = -\frac{\eta}{\pi^{D-1}} \pi^\nu \tilde{z}^\nu, \tag{6}$$

where $\tilde{z} = (\tilde{x}, \tilde{p}, \tilde{\alpha}_n, \tilde{\alpha}_n)$. Since brackets for the remaining variables $\tilde{x}^\nu, \tilde{p}^\nu, \tilde{\alpha}_n^\nu, \tilde{y}^N, \pi^N$ are rather complicated, it is convenient to simplify them by means of an appropriate variable change. The change turns out to be

$$\begin{aligned}
x^\mu &= \tilde{x}^\mu + c\pi^\mu(\pi\tilde{x}), & p^\mu &= \tilde{p}^\mu + c\pi^\mu(\pi\tilde{p}), \\
\alpha_n^\mu &= \tilde{\alpha}_n^\mu + c\pi^\mu(\pi\tilde{\alpha}_n), \\
y^\mu &= \tilde{y}^\mu + c[(\pi\tilde{x})\tilde{p}^\mu - (\pi\tilde{p})\tilde{x}^\mu] + \\
ic \sum_{n=1}^{\infty} \left[\frac{1}{n} (\pi\tilde{\alpha}_{-n})\tilde{\alpha}_n^\mu + (n \leftrightarrow -n) \right] &+ (\tilde{\alpha} - \text{sector}), \\
y^{D-1} &\equiv \tilde{y}^{D-1},
\end{aligned} \tag{7}$$

where from now $(\pi\tilde{x}) \equiv \pi^\mu \tilde{x}^\mu$, and so on. The factor c is any solution of the equation $(\pi^2)c^2 + 2c - \eta(\pi^{D-1})^{-2} = 0$, thus

$$c = \frac{1}{\pi^2} \left[-1 \pm \frac{(\eta\pi^N\pi^N)^{\frac{1}{2}}}{\pi^{D-1}} \right]. \tag{8}$$

The new variables obey to the canonical brackets

$$\{x^\mu, p^\nu\} = \eta^{\mu\nu}, \quad \{y^N, \pi^M\} = \eta^{NM}, \quad \{\alpha_n^\mu, \alpha_k^\nu\} = in\eta^{\mu\nu}\delta_{n+k,0}. \tag{9}$$

Eq.(7) is invertible, an opposite change has the same form and can be obtained from Eq.(7) by means of substitution $z \leftrightarrow \tilde{z}$, $y \leftrightarrow \tilde{y}$, $c \mapsto \bar{c}$, where

$$\bar{c} = \frac{1}{\pi^2} \left[-1 \pm \pi^{D-1} (\eta \pi^N \pi^N)^{-\frac{1}{2}} \right]. \quad (10)$$

Note that a variable change which leads to Eq.(9) is not unique. For example (for the Dirac bracket which corresponds to Eq.(2)) the following simple change

$$\begin{aligned} \alpha_n^\mu &= \tilde{\alpha}_n^\mu - \pi^\mu \frac{\tilde{\alpha}_n^{D-1}}{\pi^{D-1}}, & \alpha_{-n}^\mu &\equiv \tilde{\alpha}_{-n}^\mu, \\ y^N &= \tilde{y}^N + i \sum_{n=1}^{\infty} \frac{1}{n\pi^{D-1}} (\tilde{\alpha}_{-n}^N \tilde{\alpha}_n^{D-1} + \tilde{\tilde{\alpha}}_{-n}^N \tilde{\tilde{\alpha}}_n^{D-1}), \end{aligned} \quad (11)$$

gives also the canonical brackets for the new variables. The problem is that the Virasoro constraints, being rewritten in terms of these variables, will contain products of α_n^- oscillators: $L_n \sim p^+ \alpha_n^- + \frac{1}{2}(\pi^+)^2 \sum_{k=0}^{n-1} \alpha_{n-k}^- \alpha_k^- + \dots$. It does not allow one to resolve the constraints in the light-cone gauge. In contrast, our change (7) leads to the "linearised" form of the constraints. Namely, substitution of Eqs.(6), (7) into Eq.(1) gives the expressions

$$\begin{aligned} L_n &= \frac{1}{2} \sum_{\forall k} \alpha_{n-k}^\mu \alpha_k^\mu = 0, & \bar{L}_n &= \frac{1}{2} \sum_{\forall k} \bar{\alpha}_{n-k}^\mu \bar{\alpha}_k^\mu, \\ L_0 + \bar{L}_0 &= (p^\mu)^2 + \sum_{k=1}^{\infty} (\alpha_{-k}^\mu \alpha_k^\mu + \bar{\alpha}_{-k}^\mu \bar{\alpha}_k^\mu) = 0, \end{aligned} \quad (12)$$

$$L_0 - \bar{L}_0 = \sum_{k=1}^{\infty} (\alpha_{-k}^\mu \alpha_k^\mu - \bar{\alpha}_{-k}^\mu \bar{\alpha}_k^\mu) = 0, \quad \mu = 0, 1, \dots, D-2 \quad (13)$$

which contain the variables $p^\mu, \alpha_n^\mu, \bar{\alpha}_n^\mu$ only. Now the light-cone quantization can be carried out in the standard form [7,13,14]. One imposes the gauge $x^+ = \alpha_n^+ = \bar{\alpha}_n^+ = 0$, then the variables $p^-, \alpha_n^-, \bar{\alpha}_n^-$ can be expressed through the remaining (D-3)-dimensional oscillators $\alpha_n^i, \bar{\alpha}_n^i$, $i = 1, 2, \dots, D-3$

$$\begin{aligned} p^- &= \frac{1}{2p^+} (L_0^{tr} + \bar{L}_0^{tr} - a), & \alpha_n^- &= \frac{1}{p^+} L_n^{tr}, & \bar{\alpha}_n^- &= -\frac{1}{p^+} \bar{L}_n^{tr}, \\ L_n^{tr} &= \frac{1}{2} \sum_{\forall k} \alpha_{n-k}^i \alpha_k^i, & L_0^{tr} &= \frac{1}{2} (p^i)^2 + \sum_{k=1}^{\infty} \alpha_{-k}^i \alpha_k^i. \end{aligned} \quad (14)$$

The oscillators are arranged in the normal order, the corresponding normal ordering constant a is included into the expression for p^- .

By using of Eqs.(4), (7), (14) one obtains the light-cone Poincare generators which can be presented as

$$\begin{aligned}\mathbf{P}^\mu &= \mathbf{P}_{(D-1)}^\mu + \bar{c}\pi^\mu(\pi\mathbf{P}_{(D-1)}), \\ \mathbf{J}^{\mu\nu} &= \mathbf{J}_{(D-1)}^{\mu\nu} + y^{[\mu}\pi^{\nu]}, \\ \mathbf{P}^{D-1} &= \pm\eta(\eta\pi^N\pi^N)^{-\frac{1}{2}}(\pi\mathbf{P}_{(D-1)}), \\ \mathbf{J}^{\mu D-1} &= c\pi^{D-1}\pi^\nu\mathbf{J}_{(D-1)}^{\nu\mu} + y^{[\mu}\pi^{D-1]}. \end{aligned} \quad (15)$$

The quantities $\mathbf{P}_{(D-1)}$, $\mathbf{J}_{(D-1)}$ coincide with the standard $(D-1)$ -dimensional Poincare generators of the closed string

$$\begin{aligned}\mathbf{P}_{(D-1)}^\mu &= -p^\mu, \quad \mathbf{J}_{(D-1)}^{\mu\nu} = x^{[\mu}p^{\nu]} + iS^{\mu\nu} + i\bar{S}^{\mu\nu}, \\ S^{\mu\nu} &= \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^{[\mu} \alpha_n^{\nu]}, \end{aligned} \quad (16)$$

where it is implied that Eq.(14) was substituted. Note that $-M^2 = (\mathbf{P}^\mu)^2 + \eta(\mathbf{P}^{D-1})^2 \equiv (p^\mu)^2$ from which it follows that the last from Eq.(12) actually gives the mass formula. Thus, in terms of the new variables (7), D -dimensional Poincare generators of the theory is presented through the usual $(D-1)$ -dimensional one. It makes analysis of the anomaly terms an easy task. By construction, commutators of the quantities (15) form D -dimensional Poincare algebra modulo to the terms which can arise in the process of reordering of oscillators to the normal form. The quantities (15) have the following structure: $A(y, \pi) + B(\pi)C_{(D-1)}(x, p, \alpha, \bar{\alpha})$, where $C_{(D-1)}$ represents the generators (16). Then structure of any commutator is

$$\begin{aligned}[A^1(y, \pi), A^2(y, \pi)] &+ [A(y, \pi), B(\pi)] C_{(D-1)} \\ &+ B^1(\pi)B^2(\pi) [C_{(D-1)}^1, C_{(D-1)}^2]. \end{aligned} \quad (17)$$

The first two terms can not contain of ordering ambiguities. So the only source of the anomaly can be commutators of $(D-1)$ -dimensional generators (16). The dangerous commutator is known to be $[\mathbf{J}_{(D-1)}^{i-}, \mathbf{J}_{(D-1)}^{j-}]$, which must be zero. Its manifest form is

$$\begin{aligned}[\mathbf{J}_{(D-1)}^{i-}, \mathbf{J}_{(D-1)}^{j-}] &= \frac{1}{(p^+)^2} \left[(L_0^{tr} - \bar{L}_0^{tr} + a)S^{ij} - (L_0^{tr} - \bar{L}_0^{tr} - a)\bar{S}^{ij} + \right. \\ &\quad \left. \sum_{n=1}^{\infty} \left[\frac{D-3}{12} \left(n - \frac{1}{n} \right) - 2n \right] (\alpha_{-n}^{[i} \alpha_n^{j]} + \bar{\alpha}_{-n}^{[i} \bar{\alpha}_n^{j]}) \right], \end{aligned} \quad (18)$$

which is actually zero on the constraint surface (13) and under the conditions

$$D = 27, \quad a = 2. \quad (19)$$

Note that in terms of the variables (7) the same critical dimension arises immediately in the old covariant quantization framework also, since the no-ghost theorem can be applied without modifications to Eqs.(12), (13).

Let us discuss action which reproduces the Hamiltonian system (1)-(3). It is convenient to start from the formulation in terms of the variables (7). Then the theory is specified by $(D-1)$ -dimensional Virasoro constraints (12), (13) for x^μ and by the constraint $\pi^N \pi^N = \text{const}$ for the additional vector variable. It prompts to consider action of $(D-1)$ -dimensional string with multiplet of D Θ -terms added ¹.

$$S = S_{(D-1)} - n^N \epsilon^{ab} \partial_a A_b^N - \frac{1}{\phi} (n^2 + \varepsilon). \quad (20)$$

Note that the last term can be in fact omitted, since the only which is really necessary for the present construction is the condition $n^2 \neq 0$. $U(1)^D$ gauge invariance can be used to remove all modes of A_a^N , n^N except the zero one: $A_0^N = 0$, $A_1^N(\tau, \sigma) = y^N + \pi^N \tau$, $n^N(\tau, \sigma) = \pi^N$. While the action has only manifest $(D-1)$ Poincare invariance, Eq.(15) shows that it has also hidden D -dimensional Poincare symmetry. So one expects that it can be rewritten in a manifestly D -dimensional Poincare invariant form. The relevant action is

$$S = \frac{1}{4\pi} \int d^2\sigma \left[\frac{-g^{ab}}{2\sqrt{-g}} D_a x^N D_b x^N - n^N \epsilon^{ab} \partial_a A_b^N - \frac{1}{\phi} (n^2 + \varepsilon) \right], \quad (21)$$

where $D_a x^N \equiv \partial_a x^N - \xi_a n^N$. The local symmetries are $d = 2$ reparametrizations, Weyl symmetry and the following transformations with the parameters γ , α^N , ω_a

$$\delta x^N = \gamma n^N, \quad \delta \xi_a = \partial_a \gamma, \quad \delta A_a^N = \gamma \frac{\epsilon_{ab} g^{bc}}{\sqrt{-g}} D_c x^N; \quad (22)$$

$$\delta A_a^N = \partial_a \alpha^N + \omega_a n^N, \quad \delta \phi = \frac{1}{2} \phi^2 \epsilon^{ab} \partial_a \omega_b. \quad (23)$$

Hamiltonian analysis for the theory is similar to the one presented in [6]. After partial fixation of gauge, the theory can be formulated in terms of the phase space variables $x^N(\tau, \sigma)$, $p^N(\tau, \sigma)$, y^N , π^N which are subject to the first class constraints

$$\left(p^N \pm \frac{1}{4\pi} \Pi^N_M \partial_1 x^M \right)^2 = 0,$$

¹Note that string with one Θ -term added is known to be equivalent to D -string (see [15,16] and references therein), where it can be easily taken into account in the path integral framework. It can be clue to understanding of its appearance in the theory (20).

$$\pi^N \pi^N + \varepsilon = 0, \quad \pi^N p^N = 0. \quad (24)$$

An appropriate gauge for the last constraint turns out to be

$$\pi^N x^N = 0. \quad (25)$$

The equations (24), (25) are equivalent to Eqs.(1)-(3).

To conclude, it was demonstrated that the light-cone quantization of the theory (21) is possible, in particular, requirement of absence of anomaly in the light-cone Poincare algebra leads to the critical dimension $D = 27$. There is analogy between the action (21) and D -string which can be clue for understanding of n^N -dependent part of spectrum. Let us note also that analysis of spectrum in the light-cone gauge is more complicated as compare with the standard case. In the gauge considered the manifest symmetry is $SO(D-3)$ while the massive states should fall into representations of the little group $SO(D-1)$. Similar situation arise for $D = 11$ membrane [17,18] and was analyzed in [8]. It was demonstrated that $SO(8)$ multiplets of the first massive level for the toroidal supermembrane fall actually into representations of $SO(10)$ group. We hope that the analogous consideration is applicable to $D=11$ superstring also.

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